

Axial Vector Current and Induced Pseudoscalar Coupling Constant on $\mu^-p \rightarrow n\nu_\mu\gamma$ reaction

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Abstract

The recent TRIUMF experiment for $\mu^-p \rightarrow n\nu_\mu\gamma$ gave a surprising result that the induced pseudoscalar coupling constant g_P was larger than the value obtained from $\mu^-p \rightarrow n\nu_\mu$ experiment as much as 44 %. Reexamining axial vector current on the gauge theory, we found an additional term to the matrix element of Beder and Fearing which was used to extract the g_P value from the measured photon energy spectrum. This additional term, which is self gauge invariant, plays a key role in restoring the reliability of $g_P(-0.88m_\mu^2) = 6.77g_A(0)$. Comparison with conventional approaches is also presented.

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In semileptonic weak interaction, the strong force can generally induce four couplings additional to the usual vector and axial vector couplings, i.e. weak magnetic G_M , pseudoscalar G_P , scalar G_S and tensor G_T .

The matrix element of vector and axial vector currents are given as

$$\begin{aligned}\langle N(p')|V_a^\mu(0)|N(p)\rangle &= \bar{u}(p')[G_V(q^2)\gamma^\mu + \frac{G_S(q^2)}{2m}q^\mu + G_M(q^2)\sigma^{\mu\nu}q_\nu]\frac{\tau_a}{2}u(p) \\ \langle N(p')|A_a^\mu(0)|N(p)\rangle &= \bar{u}(p')[G_A(q^2)\gamma^\mu + \frac{G_P(q^2)}{2m}q^\mu + G_T(q^2)\sigma^{\mu\nu}q_\nu]\gamma_5\frac{\tau_a}{2}u(p),\end{aligned}\quad (1)$$

where $G_A(0) = g_A(0)$, $G_M(0) = g_M(0)$, $G_V(0) = g_V(0)$ and $G_P(q^2) = (\frac{2m}{m_\mu})g_P(q^2)$ with the nucleon and muon masses, m and m_μ . τ_a is the isospin operator. G_S and G_T belong to the second class current which has a different G-parity from the first class current, and they are assumed to be absent from the muon capture to be discussed in this paper. On the basis of the PCAC (Partially Conserved Axial Current), the induced pseudoscalar coupling constant is calculated as

$$g_P(-0.88m_\mu^2) = \frac{2m m_\mu}{m_\pi^2 + 0.88m_\mu^2}g_A(0) = 6.77g_A(0). \quad (2)$$

This value is confirmed by an experiment of the ordinary muon capture (OMC) on a proton, $\mu^-p \rightarrow n\nu_\mu$ [1].

However, such kind of determination of g_P value induces 25 % uncertainty at least, because the momentum transfer is far from the pion pole. It is extremely important to obtain a precise value for g_P of the weak hadronic current, because it plays a key role in the fundamental weak interaction processes. The only way to approach to the pion pole is the radiative muon capture (RMC) on a proton, $\mu^-p \rightarrow n\nu_\mu\gamma$.

Recently, the TRIUMF group measured the RMC photon energy spectrum and extracted a surprising result [2]

$$\hat{g}_P \equiv g_P(-0.88m_\mu^2)/g_A(0) = 9.8 \pm 0.7 \pm 0.3. \quad (3)$$

It exceeds the value obtained from the OMC as much as 44%. This discrepancy is serious because the theoretical value of g_P is predicted in a fundamental manner based on the

PCAC imposed on the axial vector current and agrees with the OMC value. As long as the PCAC is assumed to be creditable, a doubt may be cast on the result of TRIUMF experiment. However, the measured photon energy spectrum seems to be reliable in view point of their enough experimental experiences in TRIUMF. In order to solve this puzzle, one has to reexamine carefully the Beder-Fearing formula [3,4] used to extract the g_P value from the measured spectrum.

The chiral perturbation calculation (ChPT) was recently carried out for OMC [5] and well reproduced the PCAC prediction, i.e. $\hat{g}_P = 6.77$. It is also consistent with the result of the heavy baryon chiral perturbation theory [6]. Therefore, we believe that the calculation based on PCAC is reliable. Since the RMC amplitude is generated merely by a minimal coupling procedure from the OMC amplitude [3], the calculation method based on PCAC might preserve the confidence even for the RMC.

The axial current coupled to the electromagnetic field can be derived in an elegant manner, i.e. a standard gauge transformation. We start from the ordinary linear σ -model given by the Lagrangian

$$\mathcal{L}_0 = \bar{\Psi}[i\gamma^\mu\partial_\mu + g(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5)]\Psi + \frac{1}{2}[(\partial_\mu\vec{\pi})^2 + (\partial_\mu\sigma)^2] - \frac{1}{2}\mu^2(\vec{\pi}^2 + \sigma^2) - \frac{\lambda}{4}(\vec{\pi}^2 + \sigma^2)^2. \quad (4)$$

Although this Lagrangian is invariant under the SU(2) isovector infinitesimal chiral and gauge transformations, $\Psi \rightarrow \Psi' = (1 + i\gamma_5\vec{\eta} \cdot \frac{\vec{\tau}}{2})\Psi$, $\bar{\Psi} \rightarrow \bar{\Psi}' = \bar{\Psi}(1 + i\gamma_5\vec{\eta} \cdot \frac{\vec{\tau}}{2})$, $\sigma \rightarrow \sigma' = \sigma + \vec{\eta} \cdot \vec{\pi}$ and $\vec{\pi} \rightarrow \vec{\pi}' = \vec{\pi} - \vec{\eta}\sigma$, it describes only a massless fermion. In order to create the pion mass, the chiral symmetry breaking term $\zeta\sigma$ should be included into \mathcal{L}_0 . Since the vacuum expectation value of the σ field does not vanish, i.e. $\langle 0|\sigma|0\rangle = f_\pi$, the σ field is shifted as $\sigma \rightarrow \tilde{\sigma} = \sigma - f_\pi$ and, then, $\langle 0|\tilde{\sigma}|0\rangle = 0$ is fulfilled. Then, the pion and sigma meson's masses can be given as $m_\pi^2 = \mu^2 + \lambda f_\pi^2$, $m_\sigma^2 = \mu^2 + 3\lambda f_\pi^2$ and $\zeta = f_\pi m_\pi^2$, and the nucleon mass is $m = -gf_\pi$.

The resulting Lagrangian without the symmetry breaking term $\zeta\sigma$ becomes

$$\mathcal{L}_0 = \bar{\Psi}[i\gamma^\mu\partial_\mu - m + g(\tilde{\sigma} + i\vec{\tau} \cdot \vec{\pi}\gamma_5)]\Psi + \frac{1}{2}[(\partial_\mu\vec{\pi})^2 + (\partial_\mu\tilde{\sigma})^2]$$

$$-\frac{m_\pi^2}{2}\vec{\pi}^2 - \frac{m_\sigma^2}{2}\tilde{\sigma}^2 - f_\pi[\lambda(\vec{\pi}^2 + \tilde{\sigma}^2) + m_\pi^2]\tilde{\sigma} - \frac{\lambda}{4}(\vec{\pi}^2 + \tilde{\sigma}^2)^2 + \text{const} . \quad (5)$$

By the relation

$$\exp(\frac{i}{f_\pi}\gamma_5\vec{\tau} \cdot \vec{\phi}) = \cos(\frac{\phi}{f_\pi}) + i\gamma_5\vec{\tau} \cdot \hat{\phi}\sin(\frac{\phi}{f_\pi}) \equiv \frac{1}{f_\pi}[\tilde{\sigma} + i\gamma_5\vec{\tau} \cdot \vec{\pi}] , \quad (6)$$

where $\hat{\phi} = \vec{\phi}/\phi$, and replacement of $\tilde{\sigma} \rightarrow \tilde{\sigma}' = \frac{m}{g} + f_\pi\cos(\frac{\phi}{f_\pi})$, the Lagrangian can be rewritten for $g = -m/f_\pi$ as

$$\mathcal{L}'_0 = \bar{\Psi}[i\gamma^\mu\partial_\mu + gf_\pi\exp(\frac{i}{f_\pi}\vec{\tau} \cdot \vec{\phi}\gamma_5)]\Psi + \frac{1}{2}(\partial_\mu\vec{\phi})^2 , \quad (7)$$

where $\mathcal{L}'_0 = \mathcal{L}_0 - \text{const}$.

It is easy to see that the Lagrangian \mathcal{L}'_0 holds a global chiral symmetry under the infinitesimal phase transformations $\Psi \rightarrow \Psi' = (1 + i\gamma_5\frac{\vec{\tau}}{2} \cdot \vec{\eta})\Psi$, $\vec{\phi} \rightarrow \vec{\phi}' = \vec{\phi} - f_\pi\vec{\eta}$. Even when the derivative is replaced by a covariant derivative, i.e. $\partial_\mu \rightarrow D_\mu = \partial_\mu - ie\epsilon_\mu$ where ϵ_μ is the photon polarization vector, \mathcal{L}'_0 remains in the global chiral symmetry, since higher order terms than e^2 and $\vec{\eta}^2$ are ignored and thus, $e\vec{\eta}$ yields negligible small quantities.

For deriving the axial current from the Lagrangian, let us first obtain the extended Euler equation by defining the action,

$$\mathcal{S} = \int d^4x \mathcal{L}(\phi_i(x), D_\mu^\pm \phi_i(x)) , \quad (8)$$

where $D_\mu^\pm = \partial_\mu \pm \kappa_\mu$. Under an infinitesimal variation $\delta\phi_i(x)$, the stationary condition yields

$$0 = \delta\mathcal{S} = \int d^4x [\frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i + \frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)}\delta(D_\mu^\pm\phi_i)] . \quad (9)$$

Since $\delta(D_\mu^\pm\phi_i) = \delta(\partial_\mu\phi_i \pm \kappa_\mu\phi_i) = \partial_\mu(\delta\phi_i) \pm \kappa_\mu\delta\phi_i$, the integrand becomes

$$\begin{aligned} & \frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i + \frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)}(\partial_\mu\delta\phi_i) \pm \kappa_\mu\frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)}\delta\phi_i \\ &= [\frac{\partial\mathcal{L}}{\partial\phi_i} - (\partial_\mu \mp \kappa_\mu)\frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)}]\delta\phi_i + \partial_\mu[\frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)}\delta\phi_i] . \end{aligned} \quad (10)$$

Thus, the extended Euler equation is found as

$$\frac{\partial\mathcal{L}}{\partial\phi_i} - D_\mu^\mp\frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)} = 0 . \quad (11)$$

Next, we derive the extended Gell-Mann-Levy equations. The variation $\delta\phi_i = -\eta_a(x)F_i^a$ changes the Lagrangian \mathcal{L} as $\mathcal{L} + \delta\mathcal{L}$, where $\delta\mathcal{L}(\phi_i, D_\mu^\pm\phi_i)$. Then, we have

$$\begin{aligned}\delta\mathcal{L} &= \frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i + \frac{\partial\mathcal{L}}{\partial(D_\mu^{(\pm)}\phi_i)}\delta(D_\mu^\pm\phi_i) \\ &= -\frac{\partial\mathcal{L}}{\partial\phi_i}\eta_a(x)F_i^a - \frac{\partial\mathcal{L}}{\partial(D_\mu^{(\pm)}\phi_i)}(D_\mu^\pm\eta_a(x))F_i^a \\ &= -[D_\mu^\mp(\frac{\partial\mathcal{L}}{\partial(D_\mu^{(\pm)}\phi_i)}F_i^a)]\eta_a(x) - [\frac{\partial\mathcal{L}}{\partial(D_\mu^{(\pm)}\phi_i)}F_i^a](D_\mu^\pm\eta_a(x)) ,\end{aligned}\quad (12)$$

where we used the extended Euler equation in (11). Defining $(\frac{\partial\mathcal{L}}{\partial(D_\mu^\pm\phi_i)}F_i^a) = A_a^\mu$, we obtain the extended Gell-Mann-Levy equations,

$$A_a^\mu = -\frac{\partial\delta\mathcal{L}}{\partial(D_\mu^\pm\eta_a)} , \quad D_\mu^\mp A_a^\mu = -\frac{\partial\delta\mathcal{L}}{\partial\eta_a} . \quad (13)$$

If $\partial_\mu\eta_a = 0$, i.e. η_a is independent of x , eq.(12) reduces to $\delta\mathcal{L} = -(\partial_\mu A_a^\mu)\eta_a$. Since $\delta\mathcal{L} = 0$ if the lagrangian holds a global symmetry under the infinitesimal variation, we have $\partial_\mu A_a^\mu = 0$. This statement is synonymous with the Noether's theorem.

Under the local transformations, $\bar{\Psi} \rightarrow \bar{\Psi}' = \bar{\Psi}(1 + i\gamma_5\frac{\vec{\tau}}{2} \cdot \vec{\eta}(x))$, $\Psi \rightarrow \Psi' = (1 + i\gamma_5\frac{\vec{\tau}}{2} \cdot \vec{\eta}(x))\Psi$, $\vec{\phi} \rightarrow \vec{\phi}' = \vec{\phi} - f_\pi\vec{\eta}(x)$, and $\partial_\mu \rightarrow D_\mu = \partial_\mu - ie\epsilon_\mu$, the Lagrangian, \mathcal{L}'_0 reads as

$$\tilde{\mathcal{L}}_0 = \bar{\Psi}[i\gamma^\mu D_\mu + gf_\pi\exp(\frac{i}{f_\pi}\gamma_5\vec{\tau} \cdot \vec{\phi})]\Psi - \bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi \cdot (D_\mu\vec{\eta}) + \frac{1}{2}(D_\mu\vec{\phi})^2 - (D^\mu\vec{\phi}) \cdot f_\pi(D_\mu\vec{\eta}) . \quad (14)$$

For this case, eq.(12) yields

$$\delta\tilde{\mathcal{L}}_0 = -\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi \cdot (D_\mu\vec{\eta}) - (D^\mu\vec{\phi}) \cdot f_\pi(D_\mu\vec{\eta}) . \quad (15)$$

Thus, by eq.(13), the axial current and its divergence are given as

$$A_a^\mu = \bar{\Psi}\gamma^\mu\gamma_5\frac{\tau_a}{2}\Psi + f_\pi D^\mu\phi_a , \quad D_\mu^{(+)}A_a^\mu = 0 . \quad (16)$$

Here, notice that $D_\mu^{(+)} = \partial_\mu + ie\epsilon_\mu$ because $D_\mu^{(-)} = D_\mu = \partial_\mu - ie\epsilon_\mu$. If we add the chiral symmetry breaking term $-\frac{m_\pi^2}{2}\vec{\phi}^2$ to \mathcal{L}'_0 , the second equation in (16) yields

$$D_\mu^{(+)}A_a^\mu = -m_\pi^2 f_\pi\phi_a , \quad (17)$$

which is the extended form of the PCAC. The same equation was also given by Adler [7]. As is seen in eq.(16), the axial coupling constant appears to be unity. However, it is well known as $g_A = 1.25$. To cure this defect, the following chiral invariant Lagrangian is added to \mathcal{L}'_0 [9],

$$\mathcal{L}_1 = C_1 \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi \cdot f_\pi \partial_\mu \vec{\phi} , \quad (18)$$

where C_1 is determined so as to give $g_A = 1.25$. Then, the axial current becomes

$$A_a^\mu = g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\tau_a}{2} \Psi + f_\pi D^\mu \phi_a , \quad (19)$$

where $g_A = 1 + C_1 f_\pi^2$.

Operating a covariant derivative $D_\mu^{(+)}$ on eq.(19) and equating it to eq.(17), we find

$$-f_\pi (D_\mu^{(+)} D^\mu + m_\pi^2) \phi_a = g_A D_\mu^{(+)} \bar{\Psi} \gamma^\mu \gamma_5 \frac{\tau_a}{2} \Psi , \quad (20)$$

i.e., in ignoring e^2 -order term,

$$f_\pi (q_\mu^2 - m_\pi^2) \phi_a = g_A [2mi \bar{\Psi} \gamma_5 \frac{\tau_a}{2} \Psi + ie \bar{\Psi} \epsilon_\mu \gamma^\mu \gamma_5 \frac{\tau_a}{2} \Psi] . \quad (21)$$

When the solution of this equation ϕ_a with a definition, $g_A/(q^2 - m_\pi^2) = -g_P/2mm_\mu$, is substituted into eq.(19), we obtain

$$\begin{aligned} A_a^\mu(x) = & \bar{\Psi}(x) [g_A \gamma^\mu \gamma_5 + \frac{g_P(q^2)}{m_\mu} q^\mu \gamma_5 - \frac{eg_P(q^2)}{m_\mu} \epsilon^\mu \gamma_5] \frac{\tau_a}{2} \Psi(x) \\ & + \frac{eg_P(q^2)}{2mm_\mu} q^\mu [\bar{\Psi}(x) \epsilon_a \gamma^\alpha \gamma_5 \frac{\tau_a}{2} \Psi(x)] , \end{aligned} \quad (22)$$

where e^2 term is ignored. The 3rd and 4th terms come from the electro-magnetic interactions with axial current in gauge invariant way. Thereby conservation of the axial current is not retained any more as in eq.(17) even if massless pion limit is taken. More detailed discussions about the role of these terms in RMC are presented below.

It may be possible to introduce another chiral invariant Lagrangian of the form [9]

$$\mathcal{L}_2 = C_2 \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi \cdot (\vec{\phi} \times \partial_\mu \vec{\phi}) , \quad (23)$$

but this one does not contribute to the axial current. This fact can be easily seen as shown below. When eq.(23) is added to the Lagrangian, the meson field is obtained by means used above in the following form instead of eq.(20),

$$\vec{\phi} = [1 + f_\pi^2 C_2^2 B^2]^{-1} [g_A \vec{B} + f_\pi C_2 g_A (\vec{B} \times \vec{B}) + f_\pi^2 C_2^2 g_A \vec{B} (\vec{B} \cdot \vec{B})] , \quad (24)$$

where $\vec{B} = -\frac{g_P}{2mm_\mu g_A f_\pi} D_\mu^{\pi(+)} (\bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi)$. Since the second term in eq.(24) vanishes, it reduces to $\vec{\phi} = g_A \vec{B}$. This is exactly the same as that given in eq.(21).

Before applying the above result to RMC, we recapitulate Fearing's model [3,4]. Using the diagrams given in Fig.1, they evaluate the relativistic amplitude of RMC on a proton as

$$M_{fi} = M_a + M_b + M_c + M_d + M_e \quad (25)$$

with

$$\begin{aligned} M_a &= -\epsilon_\alpha \bar{u}_n \Gamma^\delta(Q) u_p \cdot \bar{u}_\nu \gamma_\delta (1 - \gamma_5) \frac{\not{k} - \not{k} + m_\mu}{-2k \cdot \mu} \gamma^\alpha u_\mu , \\ M_b &= \epsilon_\alpha L_\delta \bar{u}_n \Gamma^\delta(K) \frac{\not{p} - \not{k} + m_p}{-2k \cdot p} (\gamma^\alpha - i\kappa_p \frac{\sigma^{\alpha\beta}}{2m_p} k_\beta) u_p , \\ M_c &= \epsilon_\alpha L_\delta \bar{u}_n (-i\kappa_n \frac{\sigma^{\alpha\beta}}{2m_n} k_\beta) \frac{\not{p} + \not{k} + m_n}{2k \cdot n} \Gamma^\delta(K) u_p , \\ M_d &= -\epsilon_\alpha L_\delta \bar{u}_n (\frac{2Q^\alpha + k^\alpha}{Q^2 - m_\pi^2} \frac{g_P(K^2)}{m_\mu} K^\delta \gamma_5) u_p , \\ M_e &= \epsilon_\alpha L_\delta \bar{u}_n (\frac{ig_M}{2m} \sigma^{\delta\alpha} + \frac{g_P(Q^2)}{m_\mu} \gamma_5 g^{\delta\alpha}) u_p , \end{aligned} \quad (26)$$

where

$$\Gamma^\delta(q) = g_V \gamma^\delta + \frac{ig_M}{2m} \sigma^{\delta\beta} q_\beta + g_A \gamma^\delta \gamma_5 + \frac{g_P(q^2)}{m_\mu} q^\delta \gamma_5 , \quad (27)$$

$L_\delta = \bar{u}_\nu \gamma_\delta (1 - \gamma_5) u_\mu$, $K = n - p + k$ and $Q = n - p$ with momenta of neutron, proton and photon, n, p and k , respectively. And $m \sim m_p \sim m_n$. Other constants are taken as $g_V = 1.0$, $g_A = -1.25$, $g_M = 3.71$, $\kappa_p = 1.79$ and $\kappa_n = -1.91$ [3].

As shown above the pseudoscalar(PS) coupling between pion and nucleon is used. Since their calculation up to (d) diagram turned out to be not gauge invariant, they introduced M_e

term to satisfy gauge invariance using minimal coupling scheme (MCS) on the intermediate pion momentum at the 4th term of eq.(27).

This is a very important point to be reminded in RMC if we recollect the following facts in pion photoproduction. Usually one does not need to add a gauge term in case of PS description of pion photoproduction. For pseudovector (PV) description, on the contrary, one has to introduce a gauge term, known as seagull term, to satisfy gauge invariance. However, in RMC, without a gauge term (M_e term) the PS description itself could not be gauge invariant. This is totally different from pion photoproduction. Although RMC may be approached by an inverse pion photoproduction, it has different aspects due to the pion on its off mass shell coming from the lepton current.

Therefore one usually approaches this reaction by using current-current interaction, i.e. the leptonic weak current and the hadronic vector and axial current influenced by electromagnetic interactions due to outgoing photon. Since the axial current is obtained by simple minded MCS, it cannot be guaranteed to be physically reasonable. One needs to derive carefully the axial current in external interactions as done above.

Using the axial current eq.(22), we suggest our model which is the same as Fearing's, but add a term ΔM_e term in the following way

$$M_{fi}^{our} = M_{fi} + \Delta M_e = M_a + M_b + M_c + M_d + M_e + \Delta M_e \quad (28)$$

with

$$\Delta M_e = -\epsilon_\alpha L_\delta \bar{u}_n \left(\frac{g_P(K^2)}{m_\mu} \frac{k^\delta}{2m} \gamma_5 \gamma^\alpha \right) u_p, \quad (29)$$

The M_e term, dubbed as gauge term above, is originated from the 3rd term in eq.(22). Here the momentum dependence of g_P is fixed as Q^2 to satisfy the gauge invariance of total amplitude. ΔM_e term comes from the 4th term, but modified to be self gauge invariant in the lepton-hadron spinor spaces by fixing the momentum dependence as done in M_e term. This ΔM_e term should be understood as another independent gauge term as will be made clear later on.

This term, ΔM_e , is missing in the paper by Fearing [3,4]. Accordingly, this term was not included in the previous procedure of extracting g_P value from the experimental photon energy spectrum [2].

The transition rate is given by

$$\frac{d\Gamma_{RMC}}{dk} = \frac{\alpha G^2 |\phi_\mu|^2 m_N}{(2\pi)^2} \int_{-1}^1 dy \frac{k E_\nu^2}{W_0 - k(1-y)} \frac{1}{4} \sum_{spins} |M_{fi}|^2, \quad (30)$$

where α is the fine structure constant, G is the standard weak coupling constant, $y = \hat{k} \cdot \hat{\nu}$, $k_{max} = (W_0^2 - m_n^2)/2W_0$, $E_\nu = W_0(k_{max} - k)/[W_0 - k(1 - \nu)]$, $W_0 = m_p + m_n -$ (muon binding energy) and $|\phi_\mu|^2$ is the absolute square of muon wave function averaged over the proton which is taken as a point Coulomb.

In order to compare to the experimental results, we take the following steps. For liquid hydrogen target, muon capture is dominated through the ortho and para $p\mu p$ molecular states [2,10]. Since these molecular states can be attributed to the combinations of hyperfine states of μp atomic states [10] i.e. single and triplet states, we decompose the statistical spin mixture $\frac{1}{4} \sum_{spins} |M_{fi}|^2$ into such hyperfine states by reducing 4×4 matrix elements to 2×2 spin matrix elements. At this step, we confirmed that when the ΔM_e term was not included, eq.(28) reproduced the curves given in ref. [4]. Finally, by exploiting the mixture of muonic states relevant in experiments [2], we calculate the photon energy spectrum. The count number of the photons is now expressed as

$$N = Z \frac{d\Gamma_{RMC}}{dk}. \quad (31)$$

Here Z is determined by adjusting the value of $d\Gamma_{RMC}/dk$ without ΔM_e term for $\hat{g}_P \equiv g_P(-0.88m_\mu^2)/g_A(0) = 9.8$ so as to agree with the best fit curve in ref. [2]. With this value of Z , we have to examine the case of ΔM_e included.

Our results are shown in Fig.2. The solid curve is for the spectrum obtained in ref. [2], i.e. the result without ΔM_e term for $\hat{g}_P = 9.8$. On the other hand, the dotted curve is calculated without ΔM_e term for $\hat{g}_P = 6.77$. This curve is obviously much lower than the measured spectrum. When ΔM_e term is taken into account for $\hat{g}_P = 6.77$, we obtain the

dashed curve which is very close to the solid curve. The minor discrepancy may be due to the neglect of higher order contribution and other degree of freedom such as Δ . Our result shows that ΔM_e term restores the credit of $\hat{g}_P = 6.77$.

The number of RMC photons observed for $k \geq 60 \text{ MeV}$ is 279 ± 26 and the number of those from the solid curve is 299, while our result obtained by integrating the dotted curve spectrum is 273. Since the contribution of Δ degree of freedom is known to be a few percent [4], it is not included in the present calculation. Vector mesons such as ρ and ω make also very small contributions. Higher order terms are pointed out to be insignificant [3].

The pion field actually interacts in virtual state with the nucleon and therefore the πNN form factor may be taken into account as an off-shell effect. However, the standard πNN form factor $f_{\pi NN}(q^2) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 - q^2)$ with $\Lambda^2 = m_\rho^2 + m_\pi^2$ participates through an effective $g_P(q^2)$, i.e. $\tilde{g}_P(q^2) = g_P(q^2)f_{\pi NN}(q^2)$ but gives only $4 \sim 5\%$ contribution, because the process occurs at low momentum transfer, $q^2 = -0.88m_\mu^2$.

Recently, Kirchbach and Riska [11] proposed a PV form for the pion-induced component of the axial current. The PS coupling on the induced PS term $\frac{g_P(q^2)}{m_\mu} q^\mu \gamma_5$ is replaced by PV coupling, i.e. $\frac{g_P(q^2)}{m_\mu} q^\mu (\frac{\not{q}}{2m}) \gamma_5$. Here we discuss how to describe RMC under the PV coupling scheme. To include the electromagnetic interactions on RMC one can exploit MCS on pion momenta in this PV type axial current, so that the following model can be obtained

$$M_{fi}^{PV} = M_a^{PV} + M_b^{PV} + M_c^{PV} + M_d^{PV} + M_e^{PV} \quad (32)$$

with

$$\begin{aligned} M_a^{PV} &= -\epsilon_\alpha \bar{u}_n \Gamma^\delta(Q) u_p \cdot \bar{u}_\nu \gamma_\delta (1 - \gamma_5) \frac{\not{k} - \not{k} + m_\mu}{-2k \cdot \mu} \gamma^\alpha u_\mu, \\ M_b^{PV} &= \epsilon_\alpha L_\delta \bar{u}_n \Gamma^\delta(K) \frac{\not{p} - \not{k} + m_p}{-2k \cdot p} (\gamma^\alpha - i\kappa_p \frac{\sigma^{\alpha\beta}}{2m_p} k_\beta) u_p, \\ M_c^{PV} &= \epsilon_\alpha L_\delta \bar{u}_n (-i\kappa_n \frac{\sigma^{\alpha\beta}}{2m_n} k_\beta) \frac{\not{p} + \not{k} + m_n}{2k \cdot n} \Gamma^\delta(K) u_p, \\ M_d^{PV} &= -\epsilon_\alpha L_\delta \bar{u}_n (\frac{2Q^\alpha + k^\alpha}{Q^2 - m_\pi^2} \frac{g_P(K^2)}{m_\mu} K^\delta \frac{Q}{2m} \gamma_5) u_p, \\ M_e^{PV} &= \epsilon_\alpha L_\delta \bar{u}_n (\frac{ig_M}{2m} \sigma^{\delta\alpha} + \frac{g_P(Q^2)}{m_\mu} \frac{Q}{2m} \gamma_5 g^{\delta\alpha} - \frac{g_P(K^2)}{m_\mu} \frac{K^\delta}{2m} \gamma_5 \gamma^\alpha) u_p, \end{aligned} \quad (33)$$

where

$$\Gamma^\delta(q) = g_V \gamma^\delta + \frac{ig_M}{2m} \sigma^{\delta\beta} q_\beta + g_A \gamma^\delta \gamma_5 + \frac{g_P(q^2)}{m_\mu} q^\delta \frac{\not{q}}{2m} \gamma_5 , \quad (34)$$

Here $M_{a,b,c,d}^{PV}$ amplitudes are obtained just by changing $\frac{g_P(q^2)}{m_\mu} q^\mu \gamma_5$ in eq.(27) into $\frac{g_P(q^2)}{m_\mu} q^\mu (\frac{\not{q}}{2m}) \gamma_5$ as shown in eq.(34). The M_e^{PV} term is generated by the following MCS on the momenta of eq.(34)

$$\frac{g_P(q^2)}{m_\mu} (q^\delta - e\epsilon^\delta) \frac{\not{q} - e\not{\epsilon}}{2m} \gamma_5 . \quad (35)$$

The arbitrary momentum dependence appearing here is fixed to satisfy the gauge invariance of the whole amplitudes. If we neglect the anomalous magnetic moments $\frac{\kappa_p}{2m}(\frac{\kappa_n}{2m})$ terms in eqs.(26) and (33), then one can easily show

$$\begin{aligned} M_{a,c,d}^{PV} &= M_{a,c,d} , \\ M_b^{PV} &= M_b + \epsilon_\alpha L_\delta \bar{u}_n \left(\frac{g_P(K^2)}{m_\mu} \frac{K^\delta}{2m} \gamma_5 \gamma^\alpha \right) u_p , \\ M_e^{PV} &= M_e - \epsilon_\alpha L_\delta \bar{u}_n \left(\frac{g_P(K^2)}{m_\mu} \frac{K^\delta}{2m} \gamma_5 \gamma^\alpha \right) u_p . \end{aligned} \quad (36)$$

Since the extra terms in M_b^{PV} and M_e^{PV} amplitudes are cancelled, the whole amplitudes of both models are equal to each other, i.e. $M_{fi}^{Fearing(PS)} = M_{fi}^{PV}$, if the contributions of $\frac{\kappa_p}{2m}(\frac{\kappa_n}{2m})$ are not taken into account. The contributions of $\frac{\kappa_p}{2m}(\frac{\kappa_n}{2m})$ to the photon spectrum in RMC are examined numerically and any discernible changes at this spectrum are not found. Consequently, the above PV model also cannot explain the recent RMC experimental data, but show the same results as Fearing's model. At this step, one may argue that our correction term in eq.(29) might be a double counting because it resembles the extra term at M_e^{PV} in eq.(36) and should be cancelled with that of M_b^{PV} .

However, before final conclusions about PV scheme, there is an important point on which we make emphasis. The above MCS used in eq.(35) is not the result from any fundamental theory. According to the gauge theory as we have done in the beginning, one has to use the following MCS

$$\frac{g_P(q^2)}{m_\mu}(q^\delta - e\epsilon^\delta)\frac{\not{q} + e\not{\epsilon}}{2m}\gamma_5. \quad (37)$$

to lead to the axial current of eq.(22), that was derived theoretically from the given Lagrangian. This MCS does not give the cancellation in eq.(36), but give an additional term to the above PV model, which corresponds just to the additional term we have introduced in eq.(20), although the momentum dependence is changed to satisfy the self gauge invariance.

As another attempt, the ChPT calculations of RMC have also been carried out [13,14], but the $\hat{g}_P = 6.77$ value could not be extracted. Their results are, more or less, the same as those of Fearing's calculation [3,4]. Since these calculations are based on the PV coupling, the results are nearly same as the above PV model of eq.(33). Thereby, these calculations may have to be reexamined, taking higher order terms into account. Moreover It should be noted that the ChPT can satisfy the gauge invariance but it becomes obscure if the Δ degree of freedom is taken into account.

In the present framework, our calculation shows that $\hat{g}_P = 6.77$ is reasonable for both OMC and RMC on a proton.

Acknowledgment

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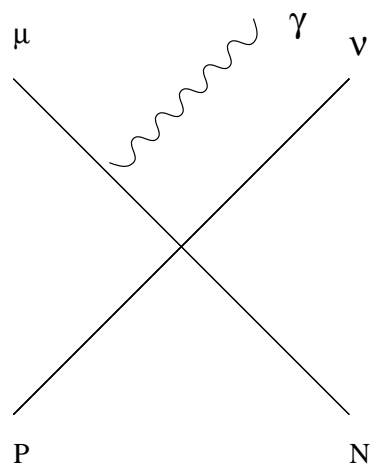
REFERENCES

- [1] G.Bardin *et al.*, Phys. Lett. **104B**, 320 (1981) and T.P.Gorringe *et al.*, Phys. Rev. Lett. **72**, 3472 (1994).
- [2] G.Jonkmans *et al.*, Phys. Rev. Lett. **77**, 4512 (1996).
- [3] H.W.Fearing, Phys. Rev. **21**, 1951(1980).
- [4] D.S.Beder and H.W.Fearing, Phys. Rev. **D35**, 2130 (1987).
- [5] H.W.Fearing, R.Lewis, N.Mobed and S.Schrer, Nucl. Phys. **A631**, 735(1998).
- [6] V.Bernard, N.Kaiser and Ulf-G. Meissner, Phys. Rev. **D50**, 6899(1994).
- [7] S.L.Adler, Phys. Rev., **139**, B1638 (1965).
- [8] F.A.Berends, A.Donnache and D.L.Weaver, Nucl. Phys. **B4**, 54(1967).
- [9] E.Kh.Akhmedov, Nucl. Phys. **A500**, 596(1989).
- [10] D.D.Bakalov, M.P.Faifman, L.I.Ponomarev and S.I.Vinitsky, Nucl. Phys. **A384**, 302(1982).
- [11] M.Kirchbach and D.O.Riska, Nucl. Phys. **A578**, 511(1994).
- [12] E.Jenkins and A.V.Manohar, Phys. Lett. **bf B255**, 558(1991).
- [13] T.Meissner, F.Myhrer and K.Kubodera, preprint, nucl-th/9707019.
- [14] S.Ando and D.P.Min, Phys. Lett. **B417**, 177(1998).

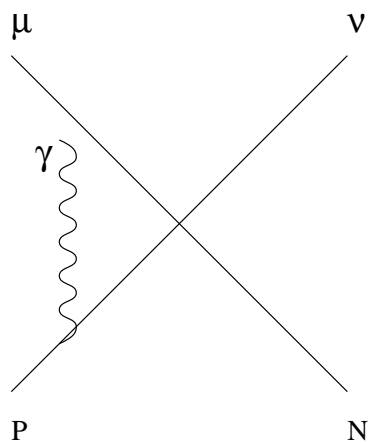
FIGURE CAPTIONS

Fig. 1. Standard diagrams describing radiative muon capture on a proton.

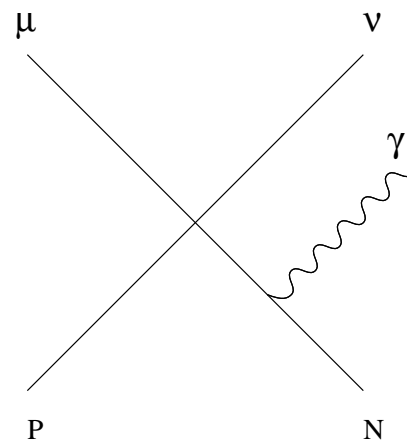
Fig. 2. Photon energy spectrum for triplet states. The solid curve, which is to reproduce the experimental data reasonably, were taken from ref. [2] , i.e. the result without ΔM_e term for $\hat{g}_P = 9.8$. The dotted curve is obtained without ΔM_e term for $\hat{g}_P = 6.77$. The dashed curve is with ΔM_e for $\hat{g}_P = 6.77$. The dot-dashed curve is calculated with ΔM_e term alone for $\hat{g}_P = 6.77$. (Figure of direct comparison with the experimental data is not presented here because of some problems in PS file transform. Please contact to the authors for more informations on our results)



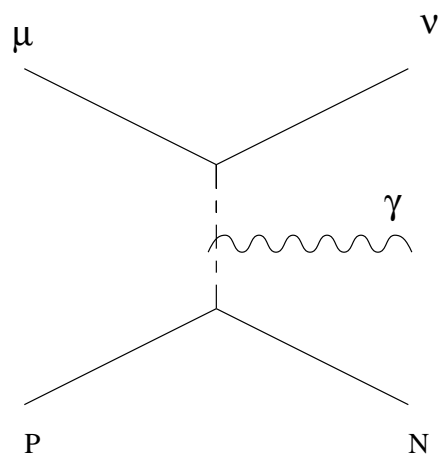
(a)



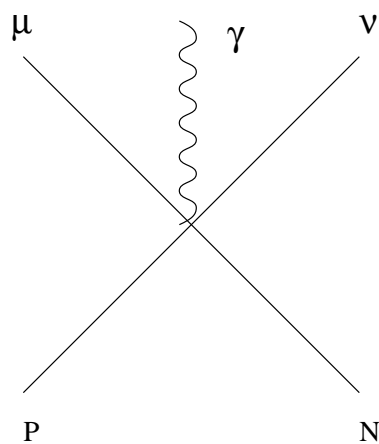
(b)



(c)



(d)



(e)

